The Time-Evolution of Property Casualty Insurance Liabilities

Stephen Mildenhall

IMA, Minneapolis, MN
January 31, 2003

Overview

1. Coverages, accident years
2. Report and settlement lags
3. Paid loss process and desired decomposition
4. Aggregate loss model
5. Poisson decomposition
6. Contagion
7. Negative multinomial descomposition

8. Average costs vs settlement lag

9. Markov model of claim complexity

10. Combined results

IMA Talk

Jan 31, 2002

C

Overview

1. Coverage
2. Accident Years
3. Report + Settlement Lags
4. Paid loss process
5. Desired Decomposition
6. Aggregate Loss Model
7. Poisson decomposition
8. Contagion & Negative Binomial distribution
9. Negative multinomial distribution

---

10. Average cost vs settlement Lag
11. Markov model of claim complexity
12. Combined frequency / severity model.

1. Coverages / Scene

> Home, auto, WC, GL, AL, aircraft, terrorism ...

> Not life, pensions

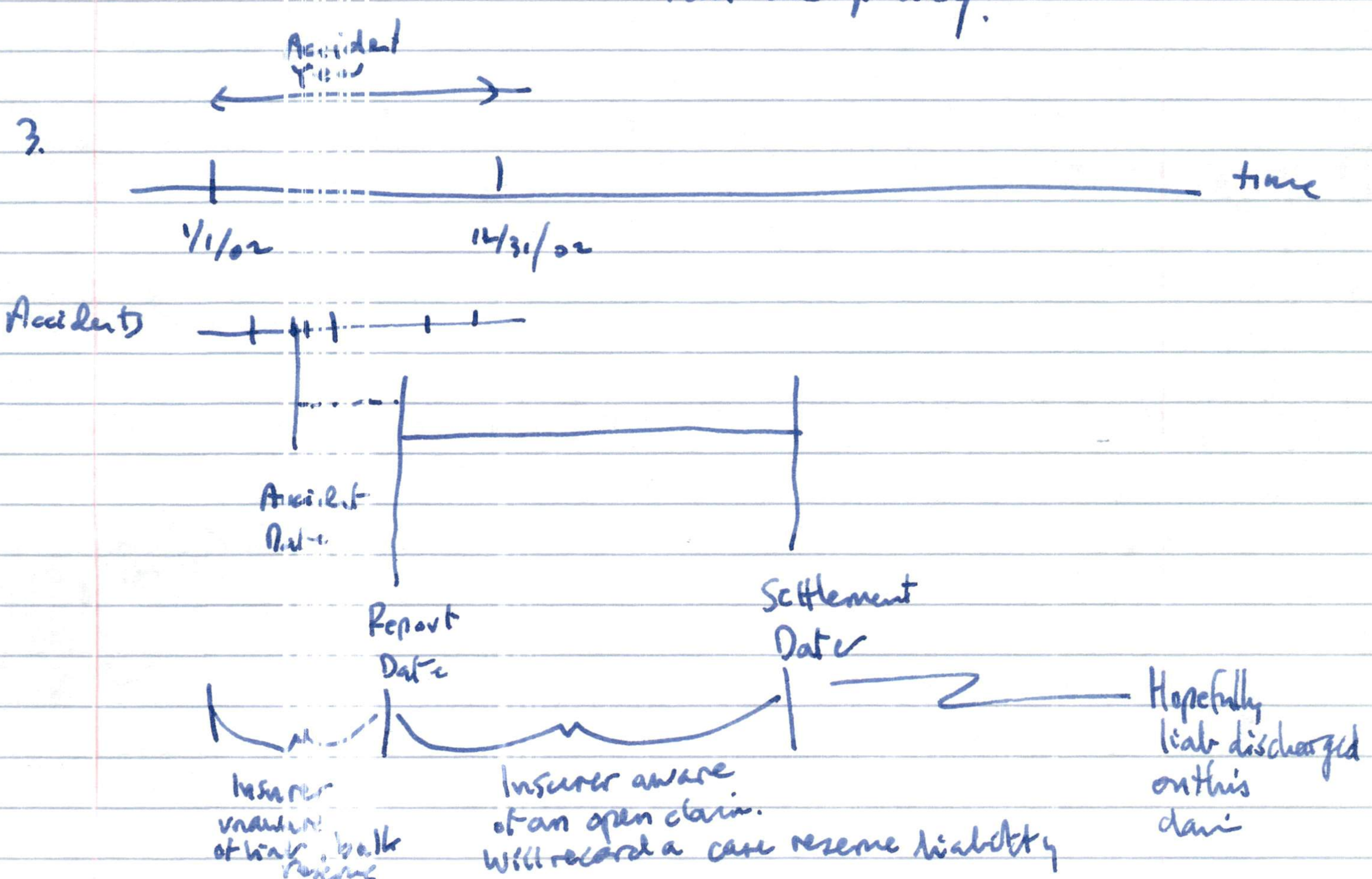
CAS vs SOA
~3000 ~15000 members

2. Accident Years, Report, Settlement Lags

Accident Year view : gather all financial implications of loss events which occur in a given year, the "accident" year

: Multiple policies, multiple claims from one policy.

3.



Reasons for lags

Reporting:
- Generally not an issue
- Days to week
  - Pipeline
- Latent Hazards such as asbestos, pollution becomes more of an issue
- Here will typically ignore

Settlement:
- Discovery
- Cost estimation and damage evaluation
- Remediation
- Court proceedings
- We pay no claims you make.

≪ Occurrence forms: in some lines accident date is not well defined (Surety, med mal) and they use a report date approach ≫

Accounting: Required to post adequate reserves for known and unknown claims

known → reported → case reserves → Adjustors: mean, median, mode, etc.

unknown → unreported → Bulk reserves (IBNR)

case reserves + Bulk reserves (IBNR) = Reserves

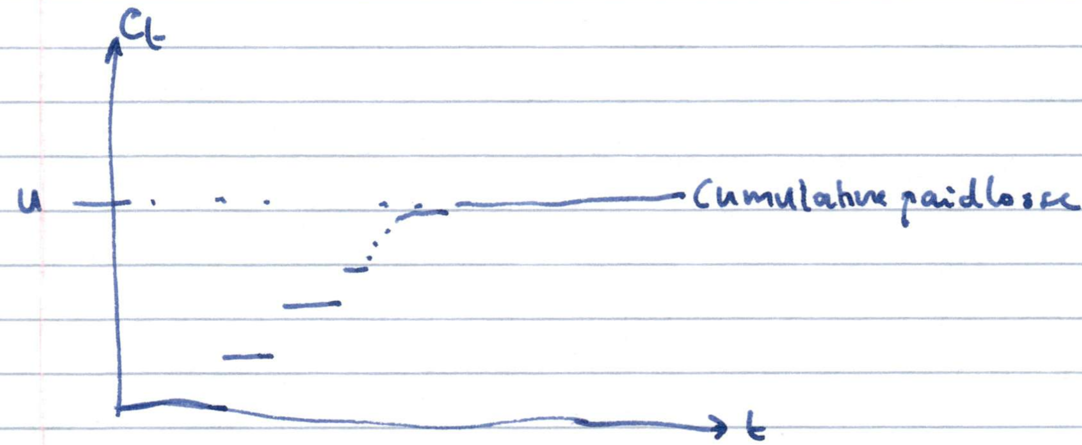
"Case development"

Reserving is #1 activity of P/C actuaries

4. The Paid Loss Process.

D_t = losses paid at time t

$C_t = \sum_{s \le t} D_s$ = Cumulative losses through time t



- C_t is a non-decreasing (no sal sub) process
- C_t converges in finite time (I hope!)

$$C_t \to U \quad \text{as } t \to \infty \quad \text{a.s.}$$

- U = ultimate loss process
- Reserves, R_t, are best estimate of unpaid liability
- Introduce probability space $(\Omega, \mathcal{F}, P)$ with an increasing filtration $\mathcal{F}_t$ on $\mathcal{F}$; $\mathcal{F}_t$ = information available at time t, then, by definition

$$R_t = E(U \mid \mathcal{F}_t) - C_t$$

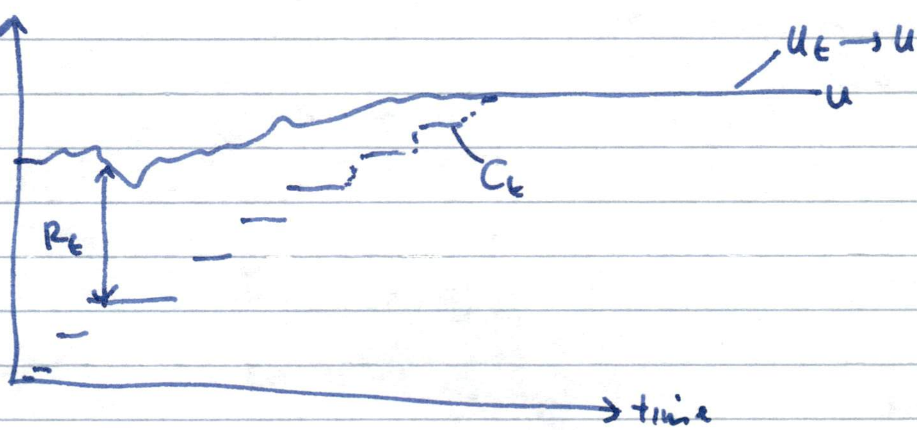
- $U_t := E(U \mid \mathcal{F}_t)$ = current estimate of ultimate losses $U_t = R_t + C_t$,

$\therefore U_t$ is a martingale

Assumption: claim settle with one payment when they close; there are no partial payments

- True for all lines except WC
- Significant portion of the market

So have a total picture of <u>Evolution of Insurance liabilities:</u>



5. Desired decomposition is

$$\boxed{U = \sum_{t \geq 0} D_t}$$

{ D_t is claims paid in the t^{th} τ rather that @ t }

6. Aggregate Loss Model

- Actuaries already have a good model for U

$$U = X_1 + \dots + X_N$$

- where

Frequency: Poisson, NB, PIG, ...

N = counting distribution giving the number of claims

Severity: Lognormal, pareto, empirical

$X_i \sim$ iid rv giving the severity of each claim.

- Claim split and not a time split.

- Very tractable decomposition because of MGF identity

$$M_X(\xi) = E(e^{\xi X}) \qquad M_N(\xi) = E(e^{\xi N})$$

- $M_U(\xi) = E(e^{\xi(X_1 + \dots + X_N)})$

$$= E(E(e^{\xi(X_1+\dots+X_N)} | N))$$

X_i iid

$$= E(M_X(\xi)^N)$$

$$\boxed{M_U(\xi) = M_N(\log M_X(\xi))}$$

- Can determine all moments of aggregate from moments of X, N.
- Can use FFT's to compute aggs where X is discrete and $E(N) \sim 100$.
- $E(U) = E(N)E(X)$; $Var(U) = E(N)Var(X) + Var(N)E(X)^2$

- Claim-by-claim view as opposed to an evolution over time view; but so nice we want to be consistent w/ approach.

7. Poisson Decomposition of the Aggregate Loss Model.

- $N \sim \text{Poisson}(n)$ $\qquad M_N(\zeta) = \exp(n(e^{\zeta} - 1))$

- Suppose that π_t of claims are reported at time t,

$$\sum_{t \geq 0} \pi_t = 1.$$

- Define D_t to be compound Poisson with frequency component $\sim \text{Poisson}(\pi_t n)$ and same severity component X.

- $$M_{D_1 t \ldots}(\zeta) = \prod_t \exp(\pi_t n (M_X(\zeta) - 1))$$

$$= \exp\left(n\left(\sum_t \pi_t M_X(\zeta) - 1\right)\right)$$

$$= M_U(\zeta)$$

$\Rightarrow$ Have a decomposition

$$U = D_1 + D_2 + \cdots$$

exactly as we wanted with

- D_i <u>indep</u> CPoissons!

- Can adjust X's to X' and you get a similar result with mixtures:

If F_i = distribution function of L_i then

$$CP\begin{bmatrix} \text{Freq} = \text{Poisson}(n) \\ \text{Sev} = \text{mixture}\{\pi_i F_i\} \end{bmatrix} = \underset{\text{Indep Sum}}{\bigoplus} CP\begin{bmatrix} \text{Freq} = \text{Poisson}(\pi_i n) \\ \text{Sev} = F_i \end{bmatrix}$$

__Example__ $\qquad L \equiv 1$ $\qquad$ (all claims cost £1; only interested in frequency uncertainty)

$N \sim \text{Poisson}(n)$ $\qquad p + q = 1$ $\qquad I = \begin{cases} 1 & p \\ 0 & q \end{cases}$

$D_1 = \text{Poisson}(pn)$

$D_2 = \text{Poisson}(qn)$ $\qquad I = 1 \Rightarrow$ settles at time 1.

Can write $\quad D_1 = I_1 + \cdots + I_N$

$\qquad\qquad D_2 = (1-I_1) + \cdots + (1-I_N) = N - D_1$

then $D_1 + D_2 = N$. $\quad D_i$'s are independent.
[Per MGF argument! Also easy to compute they are uncorrelated ... condition on N]

$E(D_1) = np \qquad E(D_2) = nq$

$$E(D_1 D_2) = E\,E\left((I_1 + \cdots + I_N)((1-I_1) + \cdots + (1-I_N)) \mid N \right)$$

$$= E\left(\sum_{i \neq j} E(I_i) E(1-I_j) \right) = E(N^2 - N)pq$$

$$= n^2 pq = E(D_1)E(D_2) \Rightarrow \text{Uncorrelated.}$$

$\text{Var} = E(N^2) - E(N)^2$

$n = E(N^2) - n^2$

$\Rightarrow E(N^2) = n^2 + n$

8. <u>What will actually happen?</u> <u>Contagion & the Neg Bin σ</u>:

Population of insureds is exposed to common shocks:

- Level of economic activity
- Gas prices (auto)
- Weather, snow storms etc.
- Fads and trends (dot com boom/bust, D+O)
- Inflation, CPI, drug costs, ...

so expected claim count n for population is really a function $n = N(\omega)$, $\omega \in \Omega$ describing state of world. Many of the common shocks are <u>multiplicative</u>. E.g. Snow makes auto accidents $x\%$ more likely. So a model

$$N(\omega) = c(\omega)\, n \quad ; \quad \text{assume } E(c) = 1 \text{ so } E(N) = n$$

is not unreasonable to describe expected counts for the coming period.

Unbiased actuarial estimate is for n claims, but in reality will see data from a Poisson$(c(\omega)n)$ rather than a Poisson(n). This leads to correlation between D's:

$$\begin{aligned} E(D_1 D_2) &= \int E(D_1 D_2 \mid c)\, f(c)\, dc \\ &= \int (cn)^2 pq\, f(c)\, dc \\ &= n^2 pq\, E(c^2) \end{aligned}$$

So $\operatorname{cov}(D_1, D_2) = E(D_1 D_2) - E(D_1)E(D_2)$
$= n^2 pq\,(E(c^2) - E(c)^2) = n^2 pq\,\mathrm{Var}(c) \neq 0.$

The variable c is sometimes called <u>contagion</u>. If $C \sim \Gamma$ mean 1 and variance c then the ~~[illegible]~~ dist^n for N with no information becomes a <u>negative binomial</u> distribution.

$$E(N) = n \qquad Var(N) = n(1+cn) > n$$

Poisson.

Thus compound NB's are more likely than C Poissons.

[over time sb able to estimate c → go to Poisson]

9. <u>Negative Multinomial Distribution</u>

Binomial (K, p) $p+q=1$		Poisson	Negative Binomial $Q-P=1$
$M_N(\xi) =$	$(q+pe^t)^K$		$(Q-Pe^{\xi})^{-k}$
mean	Kp	$Var = mean$	kP
variance	Kpq < mean		$kPQ > kP$
			$Q = 1+P$

Define $D_t = L_1 + \dots + L_{N_t}$ $\quad N_t \sim NB(\pi_t P, k)$ $\quad (k = 1/c)$

Define multivariate distribution $U \sim NB(P,k)$ target $\Sigma D_t \neq U$ (MGF's)

$$M_{D_1 \dots D_T}(\xi_1, \dots, \xi_T) = \left(Q - \sum P\pi_i M_L(\xi_i)\right)^{-k}$$

marginals CNB's OK $\rightarrow M_{D_i} = M_{D_1 \dots}(0, \dots, \xi_i, 0, \dots) = (Q - \Sigma P_j - P_i M(\xi_i))^{-k}$ $\quad M_L(0) = 1$

marginals $\xi = 0$

$Q = 1+P$

If $U = D_1 + D_2 + \dots$ then

$$M_U(\xi) = \left(Q - \sum P\pi_t M_L(\xi)\right)^{-k}$$

$$= (Q - P M_L(\xi))^{-k}$$

So the aggregate is a CNB distribution. Have a time decomposition of an aggregate where

(i) D_t's are +vely correlated

(ii) Sum is CNB = dist of U ... what you want

Note that the distⁿ is very different to a sum of D's under indep, which would have MGF

$$\prod \left(1 + \pi_t P - \pi_t P M_L(s)\right)^{-k} .$$

PICTURE?

NEXT STEPS

- Can other common counting dist^{n}'s be used? Eg PIG?

  - Need to look at form of MGF.

$q = 1 - p$

$(q + pe^t)^k$

kp

$kqp <$ mean

$Q = 1 + P$

$(Q - Pe^t)^{-k}$

kP

$kPQ >$ mean

10-12 Markov Model of Claim Complexity

- Have assumed that severity, L, is independent of settlement lag.

- In fact, very strong evidence from many LOB's that $E(L)$ increases w/ settlement lag

⇒ Need to model L_t for losses closing at time t.

- In fact settlement lag is really a proxy for claim complexity (eg BI vs PD, lawyer involved, severity of injury ...)

- Markov model, claims transition between open cx states and finally close provides good fit to data in many cases

- Postulate that severity $L_{t,r}$ is really L_r. Effect of t reflects Δ in mix between cx as quick to close, low $ claims are eliminated from the system.

  r = complexity
  t = settlement lag

- Eg

1 period (month) Transition Matrix

$$
\begin{array}{c|cccc}
 & 1 & 2 & 3 & \text{closed} \\
\hline
cx\ 1 & 0.2 & 0.1 & 0.1 & 0.6 \\
2 & 0.2 & 0.4 & 0.2 & 0.2 \\
3 & 0.05 & 0.1 & 0.8 & 0.05 \\
\text{closed} & 0 & 0 & 0 & 1
\end{array}
$$

claims stay closed

All claims start into cx 1

Let $V = \begin{bmatrix} 0.2 & 0.1 & 0.1 \\ 0.2 & 0.4 & 0.2 \\ 0.05 & 0.1 & 0.8 \end{bmatrix}$

IMA Example

One period transition matrix

V	1	2	3	Closed
	0.115	0.302	0.053	0.530028
	0.064	0.171	0.067	0.698
	0.000	0.078	0.847	0.075
	0.000	0.000	0.000	1.000

I		
1	0	0
0	1	0
0	0	1

W		
0.530	0	0
0	0.698	0
0	0	0.075

Term Dist	**0.033471437**	**0.507772**	**0.458756743**	= rescaled e'vector corresponding to largest e'value
Numerical	0.033471437	0.507772	0.458756743	

$U=W^{-1}VW$		
0.114656	0.397585	0.007499
0.048851	0.1708	0.007216
0	0.729118	0.847211

$T=(I-V)^{-1}$				Total
	1.164542931	0.482513	0.618309455	2.26536574
	0.094201646	1.297263	0.604861879	1.99632689
	0.048143566	0.662991	6.854114051	7.56524905

Dist of closing cx

TW		
0.617240872	0.33657	0.04618962
0.049929551	0.904885	0.045185045
0.025517459	0.462459	0.512023426

v_0 TW	0.617240872	0.33657	0.04618962

Eigenvectros of U = WC

WC		
-0.257592	-0.192367	**0.005412**
0.609014	-0.642288	**0.082105**
-0.002904	0.010672	**0.07418**

eig(V)	Values		Vectors, C		
	-0.0008408	0	-0.48599625	-0.3629378	0.010211
	0.277763893	0	0.873095895	-0.9207974	0.117708
	0.855745061	0	-0.03887415	0.14285758	0.992996

Scaled E'vectors

-0.739106	0.23346	0.033471
1.747438	0.779491	0.507772
-0.008332	-0.012952	0.458757

E(occ)	1.16454293	0.482513	0.618309319	2.26536559

Cls cx Dist	0.617240871	0.336569	0.04618961

Opening distribution

v_0	1	0	0
v_0 V	0.114656343	0.302109	0.053206322
v_0 V^2	0.032568604	0.090394	0.071532215
	0.009545591	0.030864	0.068426089
	0.003078716	0.013499	0.060558737
	0.001220812	0.007964	0.052379326
	0.000652005	0.005819	0.044977921
	0.000448873	0.004703	0.038532568
	0.000353823	0.003948	0.032985981
	0.000294367	0.003357	0.028230903
	0.000249565	0.002867	0.024159376
	0.000212915	0.002452	0.020674522
	0.000182021	0.002097	0.017692191
	0.000155713	0.001795	0.015140025
	0.000133237	0.001536	0.012956007
	0.000114013	0.001314	0.01108704
	9.75649E-05	0.001125	0.009487681
	8.34904E-05	0.000962	0.008119036
	7.14464E-05	0.000824	0.006947825
	6.11399E-05	0.000705	0.005945567
	5.23201E-05	0.000603	0.005087889

	n \ Cls cx	1	2	3
v_0 W	1	0.530028439	0	0
v_0 VW	2	0.060771123	0.210731	0.003974676
v_0 V^2W	3	0.017262286	0.063053	0.005343677
	4	0.005059435	0.021529	0.00511164
	5	0.001631807	0.009416	0.004523924
	6	0.000647065	0.005555	0.003912897
	7	0.000345581	0.004059	0.003359989
	8	0.000237915	0.003281	0.002878502
	9	0.000187536	0.002754	0.002464154
	10	0.000156023	0.002342	0.002108935
	11	0.000132277	0.002	0.00180478
	12	0.000112851	0.00171	0.00154445
	13	9.64761E-05	0.001463	0.001321661
	14	8.25324E-05	0.001252	0.001131006
	15	7.06193E-05	0.001071	0.000967854
	16	6.04301E-05	0.000917	0.000828236
	17	5.17122E-05	0.000784	0.000708759
	18	4.42523E-05	0.000671	0.000606517
	19	3.78686E-05	0.000574	0.000519024
	20	3.24059E-05	0.000492	0.000444152
	21	2.77312E-05	0.000421	0.000380081

Conditional Distribution by cx

1	2	3
1	0	0
0.220603	0.764968	0.014428
0.201524	0.736092	0.062383
0.159604	0.679145	0.161251
0.104795	0.604677	0.290528
0.063968	0.549207	0.386825
0.044507	0.522765	0.432728
0.037192	0.512827	0.449981
0.034694	0.509434	0.455872
0.03387	0.508313	0.457817
0.033601	0.507948	0.458451
0.033514	0.507829	0.458658
0.033485	0.50779	0.458725
0.033476	0.507778	0.458746
0.033473	0.507774	0.458753
0.033472	0.507772	0.458756
0.033472	0.507772	0.458756
0.033471	0.507772	0.458757
0.033471	0.507772	0.458757
0.033471	0.507772	0.458757
0.033471	0.507772	0.458757

V = submatrix of full transition matrix corresp. to <u>open</u> <u>cx</u>'s only.

Following facts useful & easy to verify:

(i) $T = (I - V)^{-1}$ is the mean occupancy time in each cx prior to close

Pf Eg starting in cx 1, T = time to close

$$\begin{aligned} E(T) &= \sum_t t \Pr(T = t) \\ &= \sum_t 1 - \Pr(T \le t) \\ &= \sum_t 1 - p_{1c}^{(t)} \\ &= \sum_t \sum_{j \ne c} p_{1j}^{(t)} \end{aligned}$$

$\{c\}, \{1, \ldots, \#cx\}$ are disjoint options

but $p_{1j}^{(t)} = (1,j)^{th}$ component of V^t, so result follows from $I + V + \cdots = (I - V)^{-1}$.

(ii) $\Pr_1(T = t, X_{t-1} = j)$ = Prob of closing from cx j at time t

$$= p_{1j}^{(t-1)} \cdot p_{jc}$$

So can compute bivariate distr of time to close and closing cx as

$$\text{Opening distr} \quad (*) \quad \begin{bmatrix} v_0 \\ v_0 V \\ v_0 V^2 \\ \vdots \end{bmatrix} \times \begin{bmatrix} W = \mathrm{diag}(p_{ic}) \end{bmatrix} = \begin{array}{c} \leftarrow j \rightarrow \\ \begin{bmatrix} \Pr(T = t, \\ X_{t-1} = j) \end{bmatrix} \end{array} \downarrow t$$

$\cdot\cdot \times n \qquad n \times n$

Use this expression to determine weights of L_γ's to use in each L_t.

L_t : wtd mixture of L_γ's where wts are proportional to t^{th} row of $(*)$.
(" conditional dist by cx | $T = t$)

(iii) Asymptotic mix of claims by cx is given by scaling r.W. eigenvector of V corresponding to largest evalue.
r = row (left)

=> Eventually severity stops changing, and asymptotic distribution is independent of opening cx distribution.

Pf $\quad \hat{v}_n = \dfrac{v_0 V^n}{\| v_0 V^n \|}$ then $\hat{v}_n \to$ evector associated w/ largest evalue of V.

$$\hat{v}_n V = \frac{v_0 V^{n+1}}{\| v_0 V^n \|} = \frac{v_0 V^{n+1}}{\| v_0 V^{n+1} \|} \frac{\| v_0 V^{n+1} \|}{\| v_0 V^n \|}$$

$= \hat{v}_{n+1} \cdot \lambda_n$; assume V is diag (evals), $V^n = \text{diag}(\text{eval}^n)$ and largest dominates.
$\hat{v}_{n+1}$ is a Cauchy sequence.

We are considering $v_0 V^n W$, so if $VW = WU$
$v_0 V^n W = v_0 W U^n$ and result follows.